ON THE SHAPE AND THE LENGTH OF THE CABLE DURING A DEEP-SEA TRAWLING

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paper deals mainly with the question how much cable to pay out when con- ng a trawling at a very great depth. gh a great number of trawlings have carried out in deep water no computations been published of the length of cable sary to ensure that the trawl is keeping e bottom, but the length of cable paid eems to have been estimated according evious experiences. However, the outcome her uncertain, as the shape of the cable consequently, its length, is dependent ie diameter of the cable, the velocity of hip relative to the deep water layers, and drag of the trawl. Further, as a tapered conical rope has to be used, the manner ding to which the diameter changes has at influence on the shape of the cable. efore, considering the very serious waste ne occasioned by an unsuccessful trawling, ems desirable to investigate theoretically hape of the cable during a trawling.

order to compute the shape of a cable, h is being towed in water, either without d, or carrying a load at its lower end, necessary to consider the drag of the caused by the motion relative to the r. Before examining the drag in the al case of the cable forming any angle the direction of the relative motion, we look into the simple instances of the being either perpendicular to, or parallel he direction of the motion.

e drag of a very long cylinder moving ndicularly to its axis has been measured . WIESELSBERGER.[1]) In this paper it is convenient to measure the diameter of the cable in mm, the length of the cable in m, the velocity in m/sec, and the drag in kgf. Hence, the diameter of the cable being Φ mm, the velocity of the cylinder being v m/sec, and the drag coefficient being C_p, the perpendicular drag W_p will be

$$W_p = C_p \cdot \Phi \cdot v^2 \text{ kgf/m} \qquad (1)$$

and then, according to WIESELSBERGER, $C_p = 0.055$, as the Reynold numbers occuring in the present problem have the magnitude of 10^4. In fact, the drag coefficient may vary between 0.05 and 0.06, but on the other hand, as shall be explained below, the length of the cable can be computed in such a way that the drag coefficient has a very small influence, and so it is unnecessary to take into account the variation of the drag coefficient.

The drag of a cable moving parallel to its axis was measured on board the ALBATROSS during a coring operation, the weather being at the occasion perfectly calm, and the swell being insignificant. When lowering a core-sampler weighing about 1 000 kg the tension in the cable was measured by means of a dynamometer, first during the lowering, and then during the heaving. During the heaving the dynamometer showed the tension to have increased as compared to the tension indicated during the lowering, and the difference will give twice the sum of the longitudinal drag of the cable and the drag of the core-sampler. However, as the observations are not very accurate, and the drag of the coresampler amounts to less than 10 % of the drag of the cable, no correction has been applied for the drag of the core-sampler. We have assumed the longitudinal drag, W_l, to be proportionate

[1]) C. WIESELSBERGER, Neuere Feststellungen über die ze des Flüssigkeits- und Luftwiederstandes. Phys. Z. 2, p. 321. 1921.

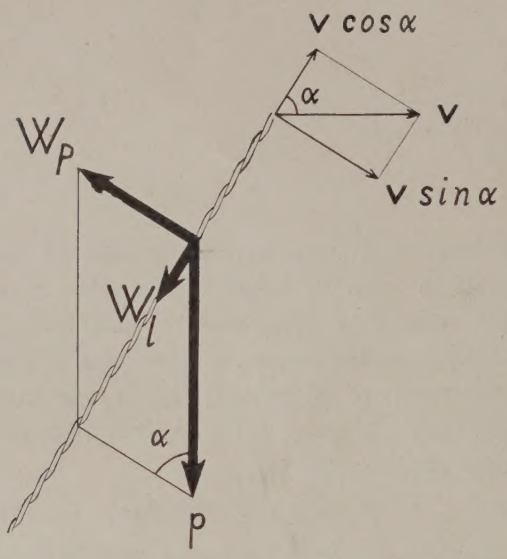


Fig. 1. Components of the drag of a cable (per unit length) forming an oblique angle with the direction of its motion, and the weight of the cable per unit length, being the forces determining the direction of a cable being towed in water at a constant velocity without carrying a load.

to the length and the diameter of the cable and the square of the velocity:

$$W_l = C_l \cdot \Phi \cdot v^2 \text{ kgf/m} \qquad (2)$$

However, it shall be pointed out that the assumption expressed by eq. (2) is not consistent with an opinion advanced by L. PRANDTL.[1])

TABLE 1 — The tension in the cable during the lowering and the heaving of a core-sampler at a velocity of 1.8 m/sec, and the longitudinal drag coefficient of the cable. Diameter of the cable 12 mm.

Length of the cable	Tension in the cable		Drag	Drag coefficient
	Lowering	Heaving		
m	kg	kg	kg	
2,000	1,750	2,270	260	0.0034
2,400	1,900	2,500	300	32
2,000	1,800	2,500	350	45
2,500	1,900	2,700	400	41
2,700	2,000	2,700.	350	33
Medium				0.0037

Proceeding to the general case of the cable forming an oblique angle with the direction

[1]) PRANDTL-TIETJENS, Hydro- und Aeromechanik, Bd. II, p. 135. Berlin 1931.

of its motion, we resolve the drag of the into two components, one being perpend to the cable, and the other one being pa to the cable. Further, we resolve the ve of the cable relative to the water into components, being respectively perpend to, and parallel to the cable. We assum component of the drag perpendicular t cable to be determined by the correspo component of the velocity according to e and we equally assume the other comp of the drag to be determined by the comp of the velocity parallel to the cable acc to eq. (2). Thus, the cable forming the a to the horizontal plane and, remaini one vertical plane, moving at the ve v m/sec in a horizontal direction belongi the same vertical plane as the cable (Fi we assume the components of the drag determined by the following expressio

$$W_p = C_p \cdot \Phi \cdot v^2 \sin^2 a \text{ kgf/m}$$
$$W_l = C_l \cdot \Phi \cdot v^2 \cos^2 a \text{ kgf/m}$$

As to the longitudinal component o drag, no attempt has been made to verif assumption. In the case of the perpend component of the drag, however, an att has been made to justify the eq. (3) by a simple experiment. Running the ship constant velocity, 50—100 m of the was paid out without carrying a load. (to the small amount of cable paid ou cable did not reach below the surface cu and so the whole of the cable moved a velocity relative to the surrounding Then the cable assumed the shape of a st line. In fact, the tension in the cable zero at the lower end, the forces determ the direction of the cable at its lower en the drag of the cable and its weight in Therefore, in its lower end the cable choose the same direction as the resulta the perpendicular drag and the weight final element of the cable, or, rather, i there choose the orientation giving a

cular drag of such a force and direction
cause the resultant of the drag and the
t of the cable to fall in the direction of
able. Then the rest of the cable will
e the same orientation, the resultant
e drag and the weight of each individual
n of the cable, and the tension of the
falling then in the same direction as
able. The weight of the cable in water
denoted p kg/m, and the angle of the
to the horizontal plane being α, then
brium demands (Fig. 1):

$$p \cdot \cos \alpha = C_p \cdot \Phi \cdot v^2 \sin^2 \alpha \qquad (5)$$

elocity of the ship, v m/sec, was measured
tting a series of drift bottles into the sea.
ting $p = 0.48$ kg/m, $\Phi = 12$ mm, and
0.055, the angle α corresponding to the
ved velocities was computed, using eq.
The angle actually formed by the cable
e horizontal plane was measured, and
angles are given in Table 2.

E 2 — Comparison between computed and measured
f the cable to the horizontal plane, the cable being
towed in water at a constant velocity

gth of cable	Velocity	α computed	α measured
55 m	0.91 m/s	49°	47°
55	0.60	66°	65°
105	0.55	69°	75°
105	0.92	49°	51°

nsidering the difficulty of making accurate
vations of the velocity of the ship, the
ment seems to justify the assumption
the perpendicular component of the drag
same as would be the case if the water
streaming perpendicularly to the cable
elocity equal to the perpendicular compo-
of the actual velocity.

en trawling in deep water the cable will
ssume the shape of a straight line. This
ious in the case of a tapered rope, because
section of the cable would try to form its
angle to the horizontal plane. As a rule,

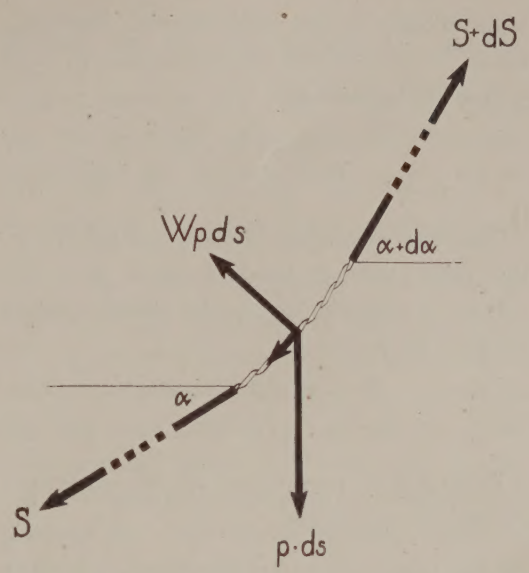


Fig. 2. Forces acting on an element of a cable moving in a horizontal direction and carrying a load at its lower end.

however, owing to the drag of the trawl, not even a cable with a uniform width would assume the shape of a straight line, though it might do so under certain conditions.

In order to investigate the shape of the cable during a trawling we will first consider a rope with a uniform width, forming a tangent to the bottom at the lower end, and there having the tension S_0 kgf. The diameter of the cable shall be denoted Φ mm, the weight of the cable in water p kg/m, the tension S kgf, and the angle of the cable to the horizontal plane α. The speed of the ship relative to the deep water layers will be denoted v m/sec. Let us consider the forces acting on an element ds of the cable (Fig. 2). There is the weight $p \cdot ds$ of the element, the longitudinal drag $W_l \cdot ds$, the perpendicular drag $W_p \cdot ds$, and the tension in the cable at both ends of the element. At the lower end of the element the tension is S, and at the other end it is $S + dS$, and it should be remembered that the two forces form between them the supplement of the angle $d\alpha$ between the tangents to the cable at both ends of the element. The forces may be resolved into components either parallel or

perpendicular to the cable element. Equating the sum of each set of components to zero we get two equations:

$$dS = p \cdot \sin \alpha \cdot ds + W_l \cdot ds \qquad (6)$$

$$S \cdot d\alpha = p \cdot \cos \alpha \cdot ds - W_p \cdot ds \qquad (7)$$

The relative velocity of the water parallel to the cable element being $v \cdot \cos \alpha$, and the velocity perpendicular to the element being $v \cdot \sin \alpha$, we have:

$$W_l = C_l \cdot \Phi \cdot v^2 \cos^2 \alpha \qquad (8)$$

$$W_p = C_p \cdot \Phi \cdot v^2 \sin^2 \alpha \qquad (9)$$

Putting these expressions into the equations (6) and (7) we obtain:

$$dS = p \cdot \sin \alpha \cdot ds + C_l \cdot \Phi \cdot v^2 \cos^2 \alpha \cdot ds \qquad (10)$$

$$S \cdot d\alpha = p \cdot \cos \alpha \cdot ds - C_p \cdot \Phi \cdot v^2 \sin^2 \alpha \cdot ds \qquad (11)$$

Supposing $p \cos \alpha - C_p \Phi v^2 \sin^2 \alpha = 0$, then $d\alpha = 0$, according to eq. (11), and the cable will not change its direction, but it will assume the shape of a straight line. We denote by β the value of α answering to this supposition. Then $\alpha = \beta$ demands $d\alpha = 0$, and *vice versa*. On the other hand, if $\alpha < \beta$, then $d\alpha > 0$, and α will approach β, whereas $\alpha > \beta$ will make $d\alpha < 0$, again causing α to approach β. Therefore, the angle β is the asymptotic value of α, giving the direction of a straight line, which the cable is approaching continually from the lower end to the upper one. In case the direction of the cable at its lower end happens to coincide with the direction of the asymptotic line, then the cable will assume the shape of a straight line through its whole length. If the cable forms a smaller angle than β to the horizontal plane at its lower end, then the cable will be concave as viewed from above, whereas it will be convex, if the angle to the horizontal at the lower end is greater than β. The direction of the cable at its lower end is governed by the forces acting on the load carried by the cable, the cable choosing the same direction as the resultant of the weight of the load and its drag. The weight of a trawl in water being muc than the drag of the trawl, the cable wi a small angle to the horizontal plane lower end during a trawling, and so the will be concave as viewed from above. difficult, however, to tell the exact di of the cable at its lower end, because th of the trawlboards deviates from the zontal direction, and, therefore, we will the cable to fall at a tangent to the at the lower end. By this assumpti arrive at slightly too high a value f minimum amount of cable to pay out i to make the trawl keep to the bottom. ever, as will be explained below, it is possible to take into consideration the direction of the cable at its lower end, pr the correct direction is known. On the hand it is obviously necessary to pay ou cable than the minimum amount, and surplus had better be regarded as a margin.

The *radii curvature* of the cable being d $R = \frac{ds}{d\alpha}$, the equations (10) and (11) n transformed into:

$$\frac{dS}{d\alpha} = (p \cdot \sin \alpha + C_l \cdot \Phi \cdot v^2 \cos^2 \alpha) \cdot R$$

$$S = (p \cdot \cos \alpha - C_p \cdot \Phi \cdot v^2 \sin^2 \alpha) \cdot R$$

By differentiating eq. (13) we get a expression for $\frac{dS}{d\alpha}$:

$$\frac{dS}{d\alpha} = (p \cdot \cos \alpha - C_p \Phi v^2 \sin^2 \alpha) \frac{dR}{d\alpha} - (p s$$

$$+ 2 C_p \Phi v^2 \sin \alpha \cdot \cos \alpha) \cdot R$$

Equating the right members of the eq (12) and (14) we get:

$$\frac{dR}{R} = \frac{2 p \sin \alpha + 2 C_p \Phi v^2 \sin \alpha \cos \alpha + C_l \Phi v^2 \cos^2 \alpha}{p \cos \alpha - C_p \Phi v^2 \sin^2 \alpha} d$$

The denominator of the right memb vanish at a certain angle $\alpha = \beta$. Cons

), where the same expression appears in ght member, it is obvious that R will finite when $\alpha = \beta$. Therefore, as ex- d above, β will be the asymptotic value nd as α is approaching β the cable will ach the shape of a straight line having me orientation as the cable would have if towed in the water at the velocity ec without carrying a load at its lower The asymptotic angle is determined by g the denominator of (15) equal ro zero.

$$\cdot \cos \beta - C_p \cdot \Phi \cdot v^2 \cdot \sin^2 \beta = 0 \qquad (16)$$

using the asymptotic angle as a para- it is possible to remove Φ (the diameter cable), p (the weight of the cable), (the velocity) from eq. (15). Trans- g eq. (16) into:

$$\frac{p}{C_p \Phi v^2} = \sin \beta \cdot \operatorname{tg} \beta \qquad (17)$$

) may be transformed into:

$$\frac{2 \sin \beta \operatorname{tg} \beta \sin \alpha + 2 \sin \alpha \cos \alpha + C_l/C_p \cdot \cos^2 \alpha}{\sin \beta \operatorname{tg} \beta \cos \alpha - \sin^2 \alpha} d\alpha \qquad (18)$$

advantage gained by using the asympto- gle β as a parameter is not purely formal.

In fact, but for the constant $\frac{C_l}{C_p}$ and the *radii curvature* of the cable at the lower end, the shape of the cable will be completely determined by the asymptotic angle. As to the constant $\frac{C_l}{C_p}$, it has a very small influence, and it can easily be determined with sufficient accuracy. The importance of the *radii curvature* R_o at the lower end of the cable will be readily understood later; it is determined by the drag of the trawl and the weight of the cable per unit length. The main thing is that the overwhelming influence of the perpendicular drag of the cable will be almost completely expressed by the asymptotic angle β. *Therefore, it is not necessary to know the drag coefficients C_l and C_p of the cable with any accuracy to determine the shape of the cable, or to compute the length of cable to pay out.* A certain error in the value of the perpendicular drag coefficient will involve nothing but a corresponding error in the value of the velocity of the trawl as computed by use of eq (17), and though the velocity of the trawl is not without interest, it is not important to know it accurately.

Integrating eq (18) we get:

$$\log \frac{R}{R_o} = \int_o^\alpha \frac{2 \sin \beta \operatorname{tg} \beta \sin \alpha + 2 \sin \alpha \cos \alpha + \frac{C_l}{C_p} \cos^2 \alpha}{(\cos \alpha - \cos \beta) \cdot (\cos \alpha + \frac{1}{\cos \beta})} d\alpha \qquad (19)$$

ving carried out the integration according to elementary methods we obtain the following ssion:

$$\log \frac{R}{R_o} = \frac{4}{3 + \cos 2\beta} \log \frac{1 - \cos \beta}{\cos \alpha - \cos \beta} + \frac{2 + 2 \cos 2\beta}{3 + \cos 2\beta} \log \frac{1 + \frac{1}{\cos \beta}}{\cos \alpha + \frac{1}{\cos \beta}}$$

$$\frac{C_l}{C_p} \left[\alpha + \frac{1 + \cos 2\beta}{\operatorname{tg} \beta (3 + \cos 2\beta)} \log \frac{\sin \frac{\beta + \alpha}{2}}{\sin \frac{\beta - \alpha}{2}} - \frac{4}{\sin \beta (3 + \cos 2\beta)} \operatorname{arc\,tg} \left(\operatorname{tg} \frac{\alpha}{2} \cdot \operatorname{tg} \frac{\beta}{2} \right) \right] \qquad (20)$$

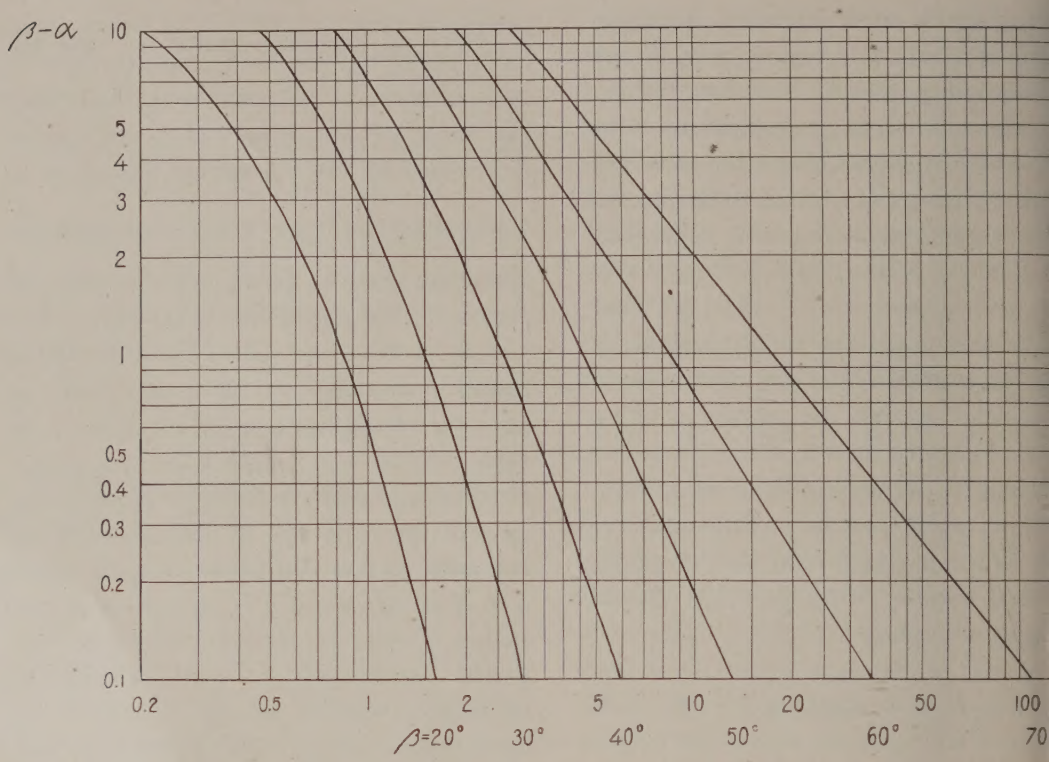


Fig. 3. Diagram representing $\beta - \alpha$ as a function of s/R_o for various values of β.

To facilitate accurate numerical computations it is convenient to transform $\cos \alpha - \cos \beta$ into a product, and for small values of $\frac{\beta - \alpha}{2}$ it is convenient to put $\sin \frac{\beta - \alpha}{2} = \frac{\beta - \alpha}{2}$. Using expression (20), and putting $\frac{C_l}{C_p} = \frac{1}{15}$, $\frac{R}{R_o}$ has been computed as a function of α for values of β ranging from 20° to 70°. Knowing the *radii curvature* it is easy to compute any quantity necessary to decide the shape of the cable, or which amount of cable to pay out at a trawling. We denote by s the length of the cable from the lower end to a point at which the cable is forming the angle α to the horizontal plane, and we further denote by z and x the vertical and the horizontal coordinates of the same point, as reckoned from the lower end of the Then:

$$\frac{s}{R_o} = \int_0^{\alpha} \frac{R}{R_o} d\alpha$$

$$\frac{z}{R_o} = \int_0^{s/R_0} \sin \alpha \cdot d\left(\frac{s}{R_o}\right)$$

$$\frac{x}{R_o} = \int_0^{s/R_0} \cos \alpha \cdot d\left(\frac{s}{R_o}\right)$$

It is further desirable to know th sion in the cable, which might be cor by eq (13), but which is more acc computed by eq (10), at least whe approaching the asymptotic angle.

$dS = p \cdot \sin \alpha \cdot ds + C_l \cdot \Phi \cdot v^2 \cdot \cos^2 \alpha \cdot$

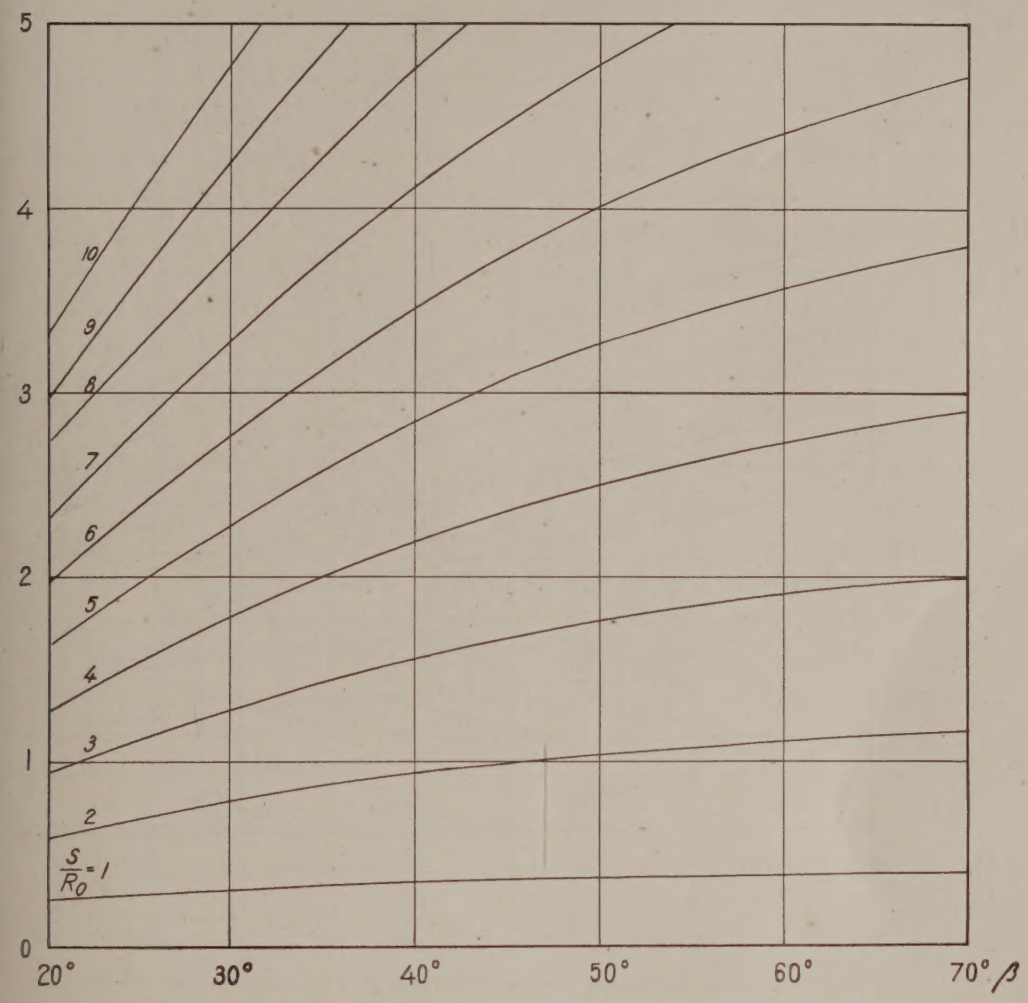


Fig. 4. Diagram representing z/R_o as a function of β for various values of s/R_o.

nsidering that $C_p \Phi v^2 : p = \cos\beta : \sin^2\beta$, $p \cdot R_o = S_o$, eq (10) may be transformed

$$\frac{dS}{S_o} = \frac{dz}{R_o} + \frac{C_l}{C_p} \cdot \frac{\cos\beta}{\sin^2\beta} \cdot \cos^2\alpha \frac{ds}{R_o} \qquad (24)$$

grating (24) we obtain:

$$1 + \frac{z}{R_o} + \frac{C_l}{C_p} \cdot \frac{\cos\beta}{\sin^2\beta} \int_0^{s/R_o} \cos^2\alpha \cdot d\left(\frac{s}{R_o}\right) \qquad (25)$$

e integrals appearing in the equations (21), (23), and (25) are carried out by numerical ration.

TABLE 3 — s/R_o as a function of α for various β

α \ β	20°	30°	40°	50°	60°	70°
5°	0.092	0.090	0.089	0.088	0.088	0.088
10°	0.206	0.188	0.183	0.180	0.179	0.177
15°	0.387	0.305	0.287	0.279	0.275	0.270
20°		0.464	0.410	0.389	0.380	0.370
25°		0.740	0.567	0.518	0.497	0.481
30°			0.794	0.679	0.634	0.606
35°			1.218	0.896	0.801	0.752
40°				1.232	1.020	0.929
45°				1.915	1.332	1.155
50°					1.848	1.464
55°					3.009	1.929
60°						2.753
65°						4.792

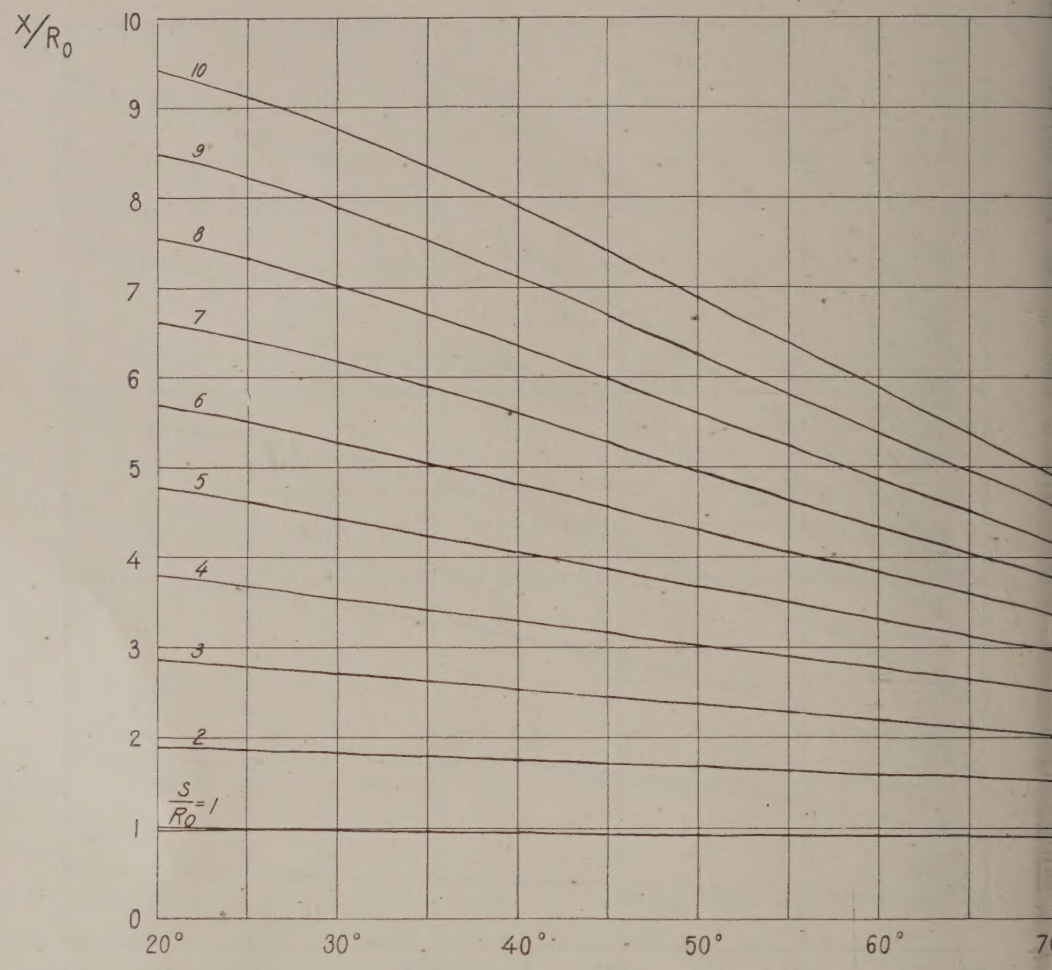


Fig. 5. Diagram representing x/R_o as a function of β for various values of s/R_o.

TABLE 4 — s/R_o as a function of $\beta - \alpha$ and β

$\beta - \alpha$ \ β	20°	30°	40°	50°	60°	70°
0.5	1.04	1.87	3.37	6.31	12.94	30.44
1	0.83	1.48	2.59	4.52	8.37	17.48
2	0.63	1.14	1.91	3.21	5.40	10.00
3	0.52	0.96	1.58	2.56	4.14	7.22
4	0.44	0.83	1.37	2.20	3.46	5.80
5	0.39	0.74	1.22	1.92	3.01	4.79
6	0.34	0.67	1.11	1.74	2.70	4.24
7	0.30	0.61	1.01	1.59	2.34	3.76
8	0.26	0.56	0.93	1.45	2.22	3.37
9	0.23	0.51	0.85	1.34	2.03	3.03
10	0.21	0.46	0.79	1.23	1.85	2.75
11	0.18	0.43	0.74	1.15	1.72	2.56
12	0.15	0.39	0.69	1.09	1.61	2.38
13	0.13	0.36	0.65	1.02	1.50	2.22
14	0.11	0.33	0.61	0.96	1.43	2.06
15	0.09	0.31	0.57	0.90	1.33	1.93
20	0.00	0.19	0.41	0.68	1.02	1.46

TABLE 5 — z/R_o as a function of s/R_o for vario

s/R_o \ β	20°	30°	40°	50°	60°
1	0.247	0.311	0.351	0.374	0.388
2	0.585	0.793	0.940	1.037	1.113
3	0.926	1.289	1.564	1.764	1.905
4	1.267	1.788	2.196	2.510	2.732
5	1.608	2.287	2.834	3.266	3.573
6	1.949	2.786	3.474	4.026	4.418
7	2.290	3.285	4.115	4.788	5.267
8	2.631	3.784	4.755	5.551	6.123
9	2.972	4.283	5.396	6.315	6.982
10	3.313	4.782	6.038	7.080	7.842
11	3.654	5.281	6.680	7.845	8.703
12	3.995	5.780	7.322	8.610	9.564
13	4.336	6.279	7.964	9.375	10.425
14	4.677	6.778	8.606	10.140	11.286
15	5.018	7.277	9.248	10.905	12.147

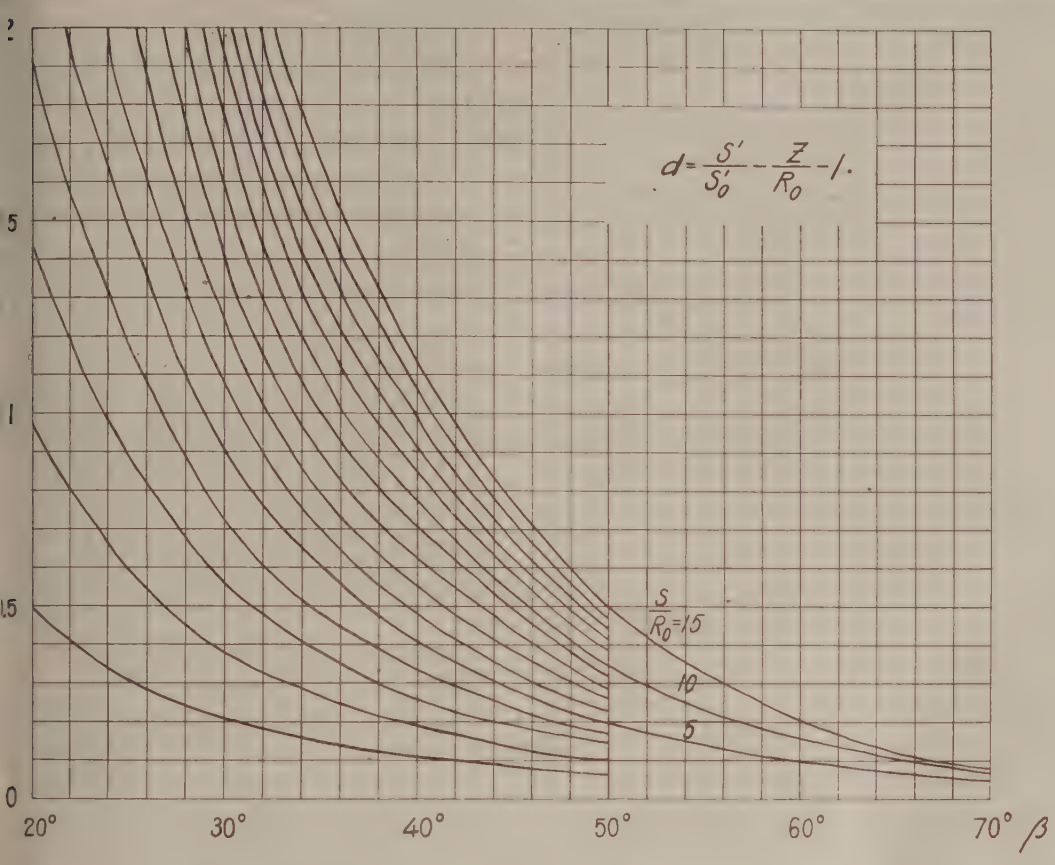


Fig. 6. Diagram representing $d = \frac{S}{S_0} - \frac{z}{R_0} - 1$ as a function of β for various values of $\frac{s}{R_0}$

TABLE 6 — x'/R_0 as a function of s/R_0 for various β

s/R_0 \ β	20°	30°	40°	50°	60°	70°
1	0.97	0.94	0.92	0.91	0.90	0.89
2	1.91	1.82	1.73	1.66	1.59	1.54
3	2.85	2.69	2.51	2.36	2.20	2.04
4	3.79	3.56	3.28	3.01	2.76	2.54
5	4.73	4.43	4.06	3.67	3.31	2.97
6	5.67	5.30	4.82	4.32	3.84	3.38
7	6.61	6.16	5.59	4.97	4.36	3.78
8	7.55	7.03	6.35	5.62	4.89	4.17
9	8.49	7.90	7.12	6.27	5.41	4.56
10	9.43	8.76	7.89	6.91	5.92	4.93
11	10.37	9.63	8.66	7.56	6.43	5.31
12	11.31	10.50	9.42	8.21	6.94	5.68
13	12.25	11.36	10.19	8.85	7.45	6.04
14	13.19	12.23	10.95	9.50	7.96	6.41
15	14.13	13.10	11.72	10.14	8.47	6.77

TABLE 7 — $d = \frac{S}{S_0} - \frac{z}{R_0} - 1$ as a function of $\frac{s}{R_0}$ for various β

β \ s/R_0	20°	30°	40°	50°	60°	70°
1	0.50	0.21	0.11	0.06	0.03	0.02
2	0.97	0.38	0.19	0.10	0.06	0.03
3	1.45	0.56	0.26	0.14	0.07	0.04
4	1.92	0.73	0.34	0.17	0.09	0.04
5	2.39	0.91	0.41	0.20	0.10	0.05
6	2.87	1.08	0.48	0.23	0.11	0.05
7	3.34	1.25	0.56	0.26	0.13	0.06
8	3.81	1.43	0.63	0.29	0.14	0.06
9	4.29	1.60	0.71	0.32	0.15	0.07
10	4.76	1.77	0.78	0.35	0.16	0.07
11	5.23	1.95	0.85	0.38	0.17	0.07
12	5.71	2.12	0.92	0.41	0.18	0.07
13	6.18	2.29	1.00	0.44	0.19	0.08
14	6.65	2.47	1.07	0.47	0.20	0.08
15	7.13	2.64	1.14	0.50	0.21	0.08

Examination of the trawling cable of the ALBATROSS

Before starting a cruise during which trawling in deep water is to be carried out, diagrams should be prepared, giving the depth of the trawl as a function of the angle formed by the cable to the horizontal plane at its upper end for each 1,000 m of cable paid out. The general solution of the present problem, advanced in the preceeding pages, is intended to serve as a basis for the making of such diagrams.

To facilitate the computations, it is advisable to represent the Tables 4—7 in diagrams. Then $\beta - \alpha$ is represented as a function of $\frac{s}{R_o}$ for various values of β, using a logaritmic scale (Fig. 3), because $\log (\beta - \alpha)$ is a nearly linear function of $\log \frac{s}{R_o}$ for small values of $\beta - \alpha$. Further, $\frac{z}{R_o}$ and $\frac{x}{R_o}$ are represented as functions of β for various values of $\frac{s}{R_o}$ (Figs. 4 and 5), because, keeping β a constant, the two quantities are nearly linear functions of $\frac{s}{R_o}$, provided $\frac{s}{R_o} > 1$, and so interpolations are easy. The quantity $d = \frac{S}{S_o} - \frac{z}{R_o} - 1$ is represented as a function of $\frac{s}{R_o}$ for various valùes of β (Fig. 6).

The trawling cable of the ALBATROSS had a length of 8,000 m, though, sometimes, it had to be extended by shackling another wire rope to its lower end. The cable was a tapered one and consisted of three sections as tabulated below.

TABLE 8 — Dimensions of the trawling cable of the ALBATROSS

Section	Diameter	Weight in water	Length
I.....	12 mm	0.48 kg/m	3,000 m
II.....	16	0.85	2,830
III.....	20	1.33	2,170

The tension in the cable at the lower e determined by the drag of the trawl. the trawl is running in contact with the bo the drag will be increased as compared t drag in water, and as, incidentally, part cable will trail on the bottom, the tensi the cable will be further increased. therefore, impossible to know the exact te in the cable at the lower end, or at the where the cable is leaving the bottom. quite sufficient, however, to know the mini value of the tension, i. e. the tension i cable at the lower end when the tra running in water without touching the bo In fact, supposing the amount of cable out to be such as to make the trawl immediately above the bottom, without t ing it, then a few metres more paid ou make the trawl touch the bottom. The is true, the tension in the cable will inc owing to the increased resistance raised t trawl, and the cable will be stretched a more than earlier, i. e. the angle of the to the horizontal plane will increase i vicinity of the bottom and decrease slight the rest of the cable. However, obviousl trawl will not leave the bottom, but intimity of the contact between the traw the bottom (as measured by the resis raised to the trawl boards) on the one and the angle of the cable to the horiz plane on the other hand, will regulate other automatically. In fact, supposing trawl boards to have gone down into the so as to raise a considerable resistance, the increased tension will cause the ca choose a steeper direction in the vicini the bottom, and then the cable will be to clear the trawl boards from the mud a consequence, the tension will diminish and normal conditions will be restored there is a margin between the minimum l of cable to pay out in order to make the only just reach the bottom, the lower e the cable then forming a small angle t

ontal plane, and the length of cable to ut in order to make the cable fall at a nt to the bottom at the lower end. In ase last mentioned the tension in the will be larger than the drag of the trawl running without touching the bottom. fore, we will assume the cable to fall at gent to the bottom at the lower end, the n being equal to the drag of the trawl not in contact with the bottom, and the amount of cable paid out correspond- the proposed assumption, the trawl will n contact with the bottom, and the cable orm an angle to the horizontal plane at ower end determined by the resistance l to the trawl boards by the mud.

e drag of the 30 ft trawl used on board ALBATROSS was measured at different ities, and the observations satisfy the e expression

$$S_o = 230 \cdot v^2 \qquad (26)$$

f denoting the drag, and v m/sec denoting elocity.

e depth corresponding to a certain length out has to be computed for four or five ities, and then it is possible to draw a am adequate to demands. By way of ple, the computations shall be carried out ne velocity, viz. $v = 1$ m/sec.

st we compute the asymptotic angle β ach section of the cable, using eq. (17).

E 9 — Asymptotic angle β for each section of the ng cable of the ALBATROSS at a speed of 1 m/sec

Section	Φ	β
.....	12 mm	45.°6
.....	16	51.°1
.....	20	55.°7

rther, the tension S_o at the lower end e cable is determined as the drag of the , and then the *radii curvature* R_o of the at its lower end is computed. At the ity $v = 1$ m/sec we find $S_o = 230$ kgf (eq. 26), and $R_o = S_o : p = 480$ m, the weight of the cable in water (p kg/m) being given in Table 8.

It is our intention to prepare a diagram giving the depth of the trawl as a function of the angle α at the top of the cable for each 1,000 m of cable paid out, and then we do not take into account the case when less than 3,000 m cable is being paid out.

For the top of section I, $s = 3{,}000$ m, we find $\frac{s}{R_o} = 6.25$, and then the direction of the cable at the upper end of section I, $\alpha = 45^\circ.3$, is found in Fig. 3. The corresponding $\frac{z}{R_o} = 3.98$ and $\frac{x}{R_o} = 4.75$ are found in Figs. 4 and 5 respectively. In Fig. 6 we find $d = 0.38$, and so $\frac{S}{S_o} = 5.36$. Knowing R_o and S_o we compute the coordinates and the tension at the top of section I: $z = 1{,}910$ m, $x = 2{,}280$ m, and $S = 1{,}230$ kgf.

Proceeding to section II, we assume that section I is taken away and replaced by the same sort of rope as the one comprising section II, yet without changing anything else at the lower end of section II, especially not the direction of the cable and its tension. The imagined downwards extension of section II is to be continued until the extended rope falls at a tangent to the horizontal plane, as demonstrated in Fig. 7. Denoting by s_{II} the length of the extension and by S_{II} and R_{II} the tension and the *radii curvature* at its lower end, we compute those quantities on the basis of our knowledge of the asymptotic angle for section II and the direction of the cable, and its tension, at the lower end of section II.

Knowing $\beta = 51^\circ.1$ and $\alpha = 45^\circ.3$ we find $\beta - \alpha = 5^\circ.8$, and then, according to Fig. 3, $\frac{s_{II}}{R_{II}} = 1.80$, $\frac{z_{II}}{R_{II}} = 0.92$ (Fig. 4), and $d = 0.09$

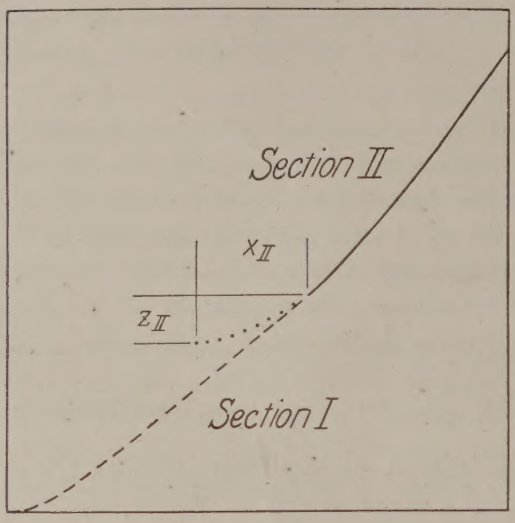


Fig. 7. By way of computing the shape of a tapered rope, the portion of the cable falling below the junction between two sections (broken curve) is imagined to be substituted by an extension (dotted curve) of the section falling above the junction (full curve).

(Table 7 or Fig. 6). Consequently, $\frac{S}{S_{II}} = \frac{z_{II}}{R_{II}} + d + 1 = 2.01$, $S_{II} = 1{,}230{:}2.01 = 610$ kgf, and $R_{II} = S_{II} : p = 720$ m. It is not necessary to compute z_{II}.

To get the direction of the cable and its coordinates at each 1,000 m of cable paid out, and for the junction between the sections II and III, we have to compute $\frac{s_{II}}{R_{II}} + \frac{1{,}000}{R_{II}}$, $\frac{s_{II}}{R_{II}} + \frac{2{,}000}{R_{II}}$, and $\frac{s_{II}}{R_{II}} + \frac{2{,}830}{R_{II}}$, the length of section II being 2,830 m. Then we find the corresponding α-values in Fig. 3. It would be feasible to extract the corresponding $\frac{z_{II}}{}$ from Fig. 4, but it is more practical to compute Δz as $\Delta z = 1{,}000 \cdot \sin \bar{\alpha}$, $\bar{\alpha}$ denoting medium of the α-values at both ends the 1,000 m in question. Similarly, $1{,}000 \cdot \cos \bar{\alpha}$.

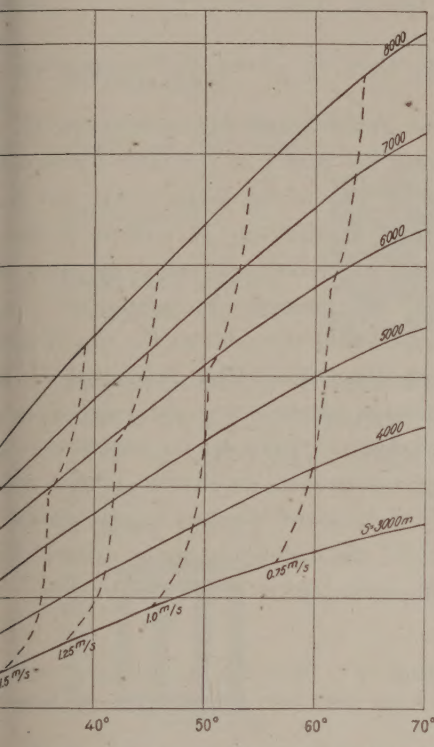
Further, it is necessary to know the t at the junction between the sections II a The total increase of the tension in sect is determined by multiplying the difference between both ends of the by p (Table 8), by multiplying the diff of the quantities d at both ends of the by S_{II}, and then adding the two pr obtained. To get the tension in the section II, the tension in its lower added.

At the lower end of section II, $d =$ and at the top of the section, $d = 0.21$ (T or Fig. 6). Further, $S_{II} = 610$ kgf, $p =$ and the height difference between both of the section = 2,130 m (Table 10). quently, the tension in the top of sect is determined as $S = 3{,}110$ kgf.

Proceeding to section III, we have $\beta =$ and, at the lower end, $\alpha = 50°.4$, hence $= 5°.3$. Then we find $\frac{s_{III}}{R_{III}} = 2.40$ (Fig. $= 1.39$ (Fig. 4), and $d = 0.10$ (Tab Consequently, $\frac{S}{S_{III}} = 2.49$, $S_{III} = 3{,}110$ $= 1{,}250$ kgf, and $R_{III} = S_{III} : p = 940$ m.

TABLE 10 — Direction and coordinates of section II

$s = 4{,}000$ m	$s = 5{,}000$ m	$s = 5{,}830$ m
$\frac{s_{II}}{R_{II}} + \frac{1{,}000}{R_{II}} = 3.19$	$\frac{s_{II}}{R_{II}} + \frac{2{,}000}{R_{II}} = 4.58$	$\frac{s_{II}}{R_{II}} + \frac{2{,}830}{R_{II}} = 5.74$
$\alpha = 49°.0$	$\alpha = 50°.0$	$\alpha = 50°.4$
$\Delta z = 730$ m	$\Delta z = 760$ m	$\Delta z = 640$ m
$z = 2{,}640$ m	$z = 3{,}400$ m	$z = 4{,}040$ m
$\Delta x = 680$ m	$\Delta x = 650$ m	$\Delta x = 530$ m
$x = 2{,}960$ m	$x = 3{,}610$ m	$x = 4{,}140$ m



. Diagram valid for the cable of the ALBATROSS trawling with a 30 ft trawl, and representing the (z km) of the trawl as a function of the angle (α) d by the cable to the horizontal when leaving the or various values of the amount of cable paid out urves). The broken curves indicate the velocity of the trawl.

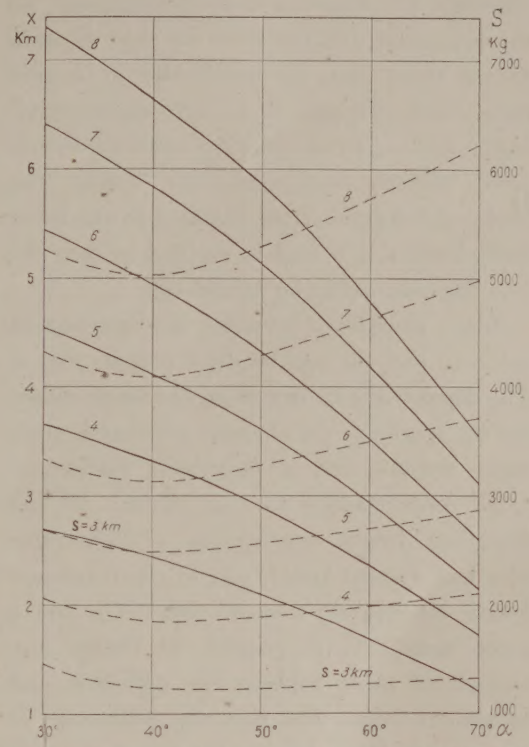


Fig. 9. Diagram valid for the cable of the ALBATROSS when trawling with a 30 ft trawl, and representing the horizontal distance (x km) from the ship to the trawl as a function of the angle (α) formed by the cable to the horizontal when leaving the ship, for various values of the amount of cable paid out (full curves). The broken curves indicate the tension in the cable.

e same computations having been carried or a series of values for the velocity of the , diagrams have been drawn, giving the z (Fig. 8) and the horizontal distance x 9) to the trawl as functions of the angle med by the cable to the horizontal plane leaving the ship, for each 1,000 m of cable paid out. The velocity is indicated in Fig. 8, and the tension in Fig. 9. As a rule, it is not important to know the horizontal distance from the ship to the trawl. It may happen, however, that the echo-sounder shows a sudden decrease of the depth during a trawling, displaying a steep rise likely to endanger the trawl.

TABLE 11 — Direction and coordinates of section III

s = 6,000 m	s = 7,000 m	s = 8,000 m
$\frac{s_{III}}{R_{III}} + \frac{170}{R_{III}} = 2.58$	$\frac{s_{III}}{R_{III}} + \frac{1,170}{R_{III}} = 3.65$	$\frac{s_{III}}{R_{III}} + \frac{2,170}{R_{III}} = 4.71$
$\alpha = 51.^{\circ}0$	$\alpha = 53.^{\circ}1$	$\alpha = 54.^{\circ}0$
$\Delta z = 130$ m	$\Delta z = 790$ m	$\Delta z = 800$ m
$z = 4,170$ m	$z = 4,960$ m	$z = 5,760$ m
$\Delta x = 110$ m	$\Delta x = 610$ m	$\Delta x = 590$ m
$x = 4,250$ m	$x = 4,860$ m	$x = 5,450$ m

Then the trawling has to be discontinued. As the horizontal distance from the ship the ship to the trawl may be considerable at a very great depth, it is possible to carry on the trawling for some time after the rise has been discovered, without endangering the trawl. The horizontal distance from the ship to the trawl being known, it is easy to decide how long the trawling may safely be carried on.

When starting a trawling the amount of cable to pay out can be derived from Fig. 8. The depth being known along the track chosen for the trawling, the diagram presents a minimum amount and a maximum amount of cable corresponding to that depth. To the minimum length corresponds a high value of α and a small speed, and to the maximum length corresponds a small value of α and a great speed. It is possible to choose any amount of cable between the minimum and maximum values, and then the inclination of the cable is determined by the length of cable decided upon. When the cable has been paid out, the speed of the ship has to be regulated carefully in such a way that the cable keeps to the predetermined inclination. The angle α has to be measured at close intervals in order that deviations from the correct value shall be discovered at once and the speed of the ship adjusted accordingly.

As pointed out above, it is not necessary to assume the cable to fall at a tangent with the bottom during the trawling, but it is quite possible to allow the cable to form a small angle to the horizontal at its lower end. We will assume the angle in question to be α_o, and we shall see how the amount of cable to pay out is affected. Disregarding the change of tension from $\alpha = 0$ to $\alpha = \alpha_o$, we e

the values of $\frac{s}{R_o}$ and $\frac{z}{R_o}$ corresponding

from Table 3 and Fig. 4 respectively. posing the part s of the cable between and $\alpha = \alpha_o$ to be taken away, and in instead in the top of section I, the height difference between both ends cable will increase by $s \cdot \sin 45° -$ inclination of the cable at the top of sec being about 45°. Here z denotes the difference between $\alpha = 0$ and $\alpha = \alpha_0$ as in Table 12. In order to restore the height difference, we have to take in a ponding amount of cable, viz. ($s \cdot \sin 45°$ sin 50°, the inclination of the cable at t being about 50° at a speed of 1 m/sec.

TABLE 12 — Decrease in the amount of cable out if the cable forms the angle α_o to the horizo its lower end

α_o	$\frac{s}{R_o}$	s	$\frac{z}{R_o}$	z	$\frac{s \cdot \sin 45}{\sin 5}$
10°	0.18	86 m	0.00	0 m	80
15°	0.28	135	0.03	15	100
20°	0.40	190	0.07	35	130
25°	0.54	260	0.12	60	160
30°	0.74	355	0.23	110	180
35°	0.98	470	0.36	170	210

As demonstrated in the last colun Table 12 the amount of cable to be pai is affected very little indeed by a char the direction of the cable at its lowe The explanation is that, the *radii cur* of the cable being small at its lower en cable changes its direction very rapid the vicinity of the bottom.